Birzeit University-Faculty of Engineering
Department of Electrical and Computer Engineering
Signals and Systems, ENEE2302

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Exercise: Consider the periodic signal $x(t)$ given by the expression

$$x(t) = (2+j2)\,e^{-j30\pi t} - j3\,e^{-j20\pi t} + 5 + j3\,e^{j20\pi t} + (2-j2)\,e^{j30\pi t}$$

1. what is the average value of the signal $x(t)$.
2. Determine the expression of complex coefficient Fourier series.
3. Justify that $x(t)$ is a real signal and write the corresponding compact trigonometric fourier series representation.
4. Plot the two-sided (double-sided) amplitude and phase spectra for the signal $x(t)$.

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* Exercise Ans *

It seems an exponential fourier Series, $x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} X_n e^{jn2\pi f_0 t}$

* بما أنها سيرز، الحدود المعطاة في السؤال ناتجة عن تعويض أرقام *

① avg value $X_0 = 5$, $X_0 e^{0} = 5e^{0} = 5$

② from ex $30\pi = \omega_0 = 2\pi f_0 \Rightarrow f_1 = 15$; $f_1 = n_1 f_0$
$20\pi = \omega_0 = 2\pi f_0 \Rightarrow f_2 = 10$; $f_2 = n_2 f_0$

f_0: 5 | 10, 2 | 2, | 1 ; 5 | 15, 3 | 3, | 1

$\Rightarrow 15 = n_1 * 5 \Rightarrow n_1 = 3$
$10 = n_2 * 5 \Rightarrow n_2 = 2$

$(2-j2)e^{j30\pi t} \Rightarrow$ when $n = 3 \Rightarrow X_3 = 2 - j2$
$(2+j2)e^{-j30\pi t} \Rightarrow$ when $n = -3 \Rightarrow X_{-3} = 2 + j2$
$j3\, e^{j20\pi t} \Rightarrow$ when $n = 2 \Rightarrow X_2 = j3$.
$-j3\, e^{-j20\pi t} \Rightarrow$ when $n = -2 \Rightarrow X_{-2} = -j3$

③ $x(t) = 2e^{-j30\pi t} + 2je^{-j30\pi t} - 3je^{-j20\pi t} + 5 + 3je^{j20\pi t} + 2e^{j30\pi t} - 2je^{j30\pi t}$

$\Rightarrow \frac{2*}{2*}\left(2e^{j30\pi t} + 2e^{-j30\pi t}\right) + \frac{j2}{j2}*\left[-2je^{j30\pi t} + 2je^{-j30\pi t}\right] + \frac{j2}{j2}*\left[3je^{j20\pi t} - 3je^{-j20\pi t}\right] + 5$

$\Rightarrow 4\cos(30\pi t) + 4\sin(30\pi t) - 6\sin(20\pi t) + 5$

Real can be written as a sum of sin & cos

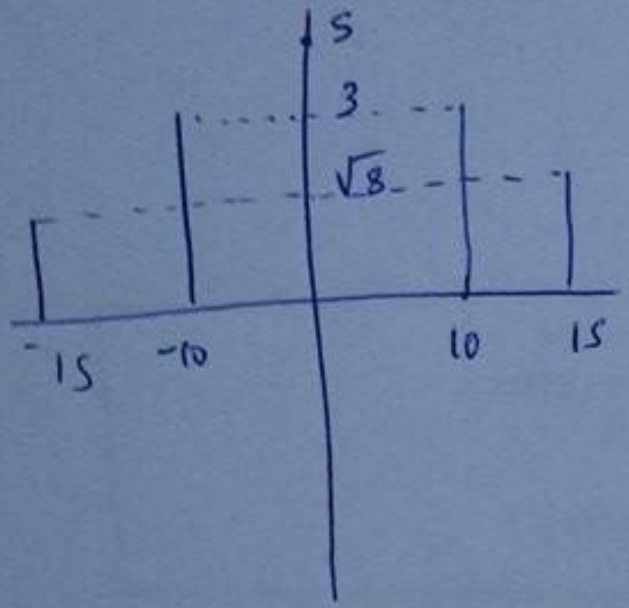
at $n = 3 \Rightarrow X_3 = \sqrt{2^2 + j2^2} = \sqrt{8} = X_{-3}$

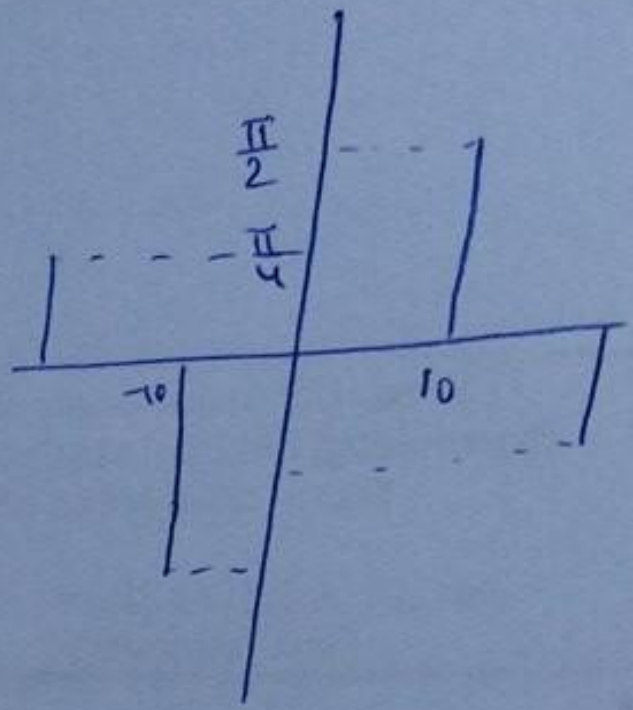
$\Theta_3 = -\Theta_{-3} = \tan^{-1}(-1) = -\frac{\pi}{4}$

at $n = 2 \Rightarrow j3 \Rightarrow X_n = 3$

$\Theta_2 = -\Theta_{-2} = \tan^{-1}\left(\frac{3}{0}\right) = \frac{\pi}{2}$

(4)





Problem #2:

A signal $x(t) = 2 + 3\cos(200\pi t)$ modulates the amplitude of a carrier signal $c(t) = 4\cos(2000\pi t)$

a. Plot the double sided spectral representation of the signal $x(t)$ and the carrier $c(t)$

b. Determine and plot the spectral representation of the modulated signal $m(t) = x(t).c(t)$

c. Determine and plot the spectral representation of the Hilbert transformed signal $x^H(t)$

(a) $X(t) = 2 + 3\cos(200\pi t) = 2 + 3\left[\frac{e^{j2\pi(100)t} + e^{-j2\pi(100)t}}{2}\right]$

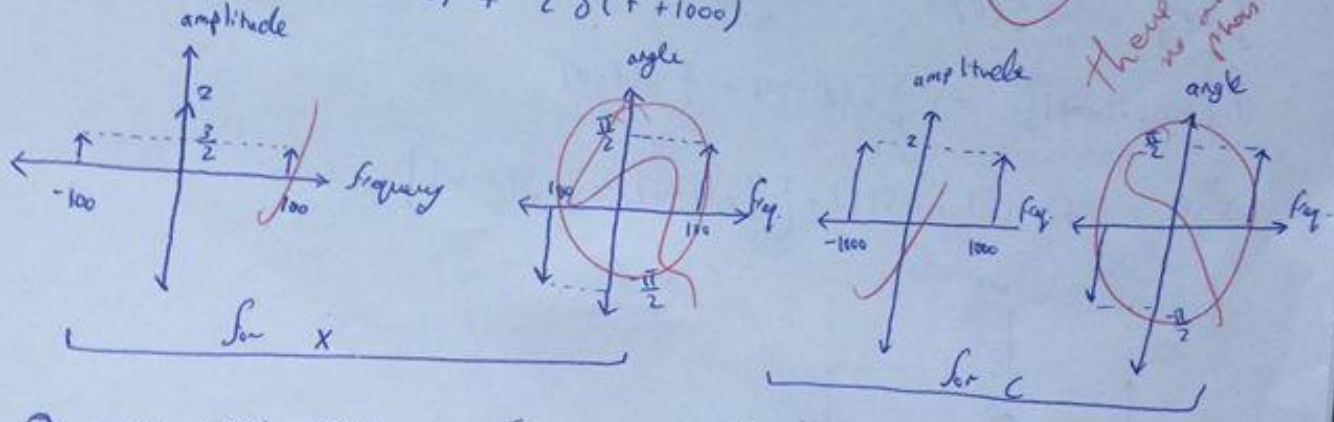
take fourier transform for both side

$X(f) = 2\delta(f) + \frac{3}{2}\delta(f-100) + \frac{3}{2}\delta(f+100)$

$C(t) = 4\cos(2000\pi t) = 4\left[\frac{e^{j2\pi(1000)t} + e^{-j2\pi(1000)t}}{2}\right]$

take fourier transform for both side

$C(f) = 2\delta(f-1000) + 2\delta(f+1000)$



(b) $m(t) = X(t).C(t) = [2 + 3\cos(200\pi t)].\ 4\cos(2000\pi t)$

$\Rightarrow 8\cos(2000\pi t) + 12\cos(200\pi t)\cos(2000\pi t)$

$\Rightarrow 8\cos(2000\pi t) + 12\left[\frac{\cos(2200\pi t) + \cos(1800\pi t)}{2}\right]$

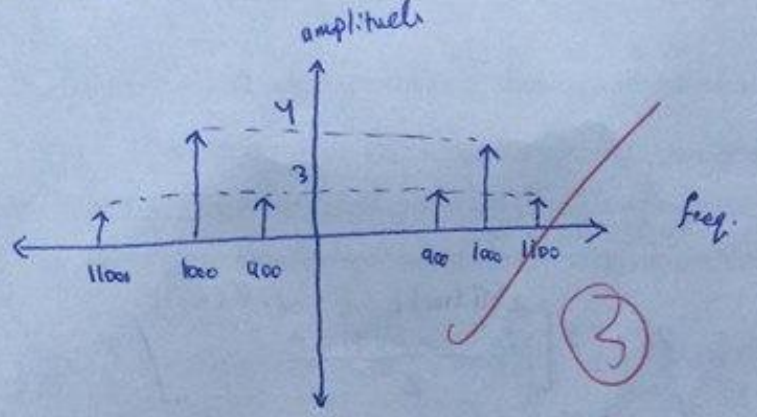
GOOD LUCK ☺

$\Rightarrow 8\cos(2000\pi t) + 6\cos(2200\pi t) + 6\cos(-1800\pi t)$

$\Rightarrow 4\left[\frac{e^{j2\pi(1000)} + e^{-j2\pi(1000)}}{2}\right] + 3\left[\frac{e^{j2\pi(1100)} + e^{-j2\pi(1100)}}{2}\right] + 3\left[\frac{e^{-j2\pi 900} + e^{j2\pi(900)}}{2}\right]$

Take Fourier Transform for both side.

$$m(f) = 4\delta(f-1000) + 4\delta(f+1000) + 3\delta(f-1100) + 3\delta(f+1100) + 3\delta(f-900) + 3\delta(f+900)$$



(3)

(c) $x^H(t) = \frac{1}{\pi t} * [2 + 3\cos(200\pi t)]$

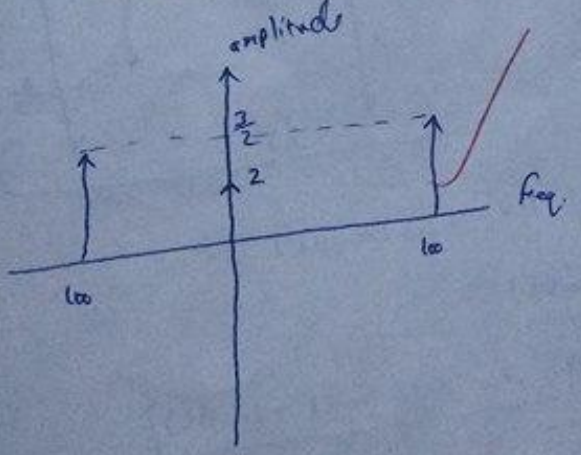
$X^H(f) = \mathcal{F}\left[\frac{1}{\pi t}\right] \mathcal{F}[2 + 3\cos(200\pi t)]$

$\mathcal{F}\left[\frac{1}{\pi t}\right] = -j\,\mathrm{sgn}(f)$

$\mathcal{F}[2] = 2\delta(f)$

$\mathcal{F}[3\cos(200\pi t)] = \frac{3}{2}\delta(f-100) + \frac{3}{2}\delta(f+100)$

$X^H(f) = -j\,\mathrm{sgn}(f) \cdot 2\delta(f) \cdot \left[\frac{3}{2}\delta(f-100) + \frac{3}{2}\delta(f+100)\right]$



-2.5

hi phase 2!